# Threshold Effects in Monetary and Fiscal Policies in a Growth Model: Assessing the Importance of the Financial System

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**Abstract:** We extend the Barro (1990) model to money financing of public expenditures, in a setup where money demand is motivated by a "cash-in-advance" constraint, in the spirit of Palivos & Yip (1995). Allowing for productive public spending yields threshold effects between long-run growth and both income taxation and seigniorage, which may reproduce recent stylized facts. Moreover, we show that, to maximize long-run growth and/or welfare, government must draw on seigniorage only or taxes only, depending on parameters, and in particular on the degree of financial development. To deal with interior solutions in which both seigniorage and taxes are used, we allow for a more general form of money demand, through transaction costs. An empirical section confirms our theoretical results.

**Keywords:** endogenous growth, threshold effects, monetary policy, fiscal policy, financial repression, financial development

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The endogenous growth theory has provided several interesting results about the impact of fiscal and monetary policies on long-run economic growth. Concerning fiscal policy, standard endogenous growth models show the existence of a threshold in the taxes to long-run growth relation, in link with the pioneer work of Barro (1990). Concerning monetary policy, most of theoretical results in endogenous growth models (*e.g.* Alogoskoufis & Van der Ploeg, 1991, Wang & Yip, 1992, Jones & Manuelli, 1993, or Palivos & Yip, 1995) conclude that it is harmful or at best neutral in the long-run, as in exogenous growth models. Empirical work is less conclusive (see, *e.g*. Levine & Renelt, 1992 and Barro, 1995). Recent work seems to confirm the existence of a negative correlation between inflation and economic growth, but only for high-inflation countries, which may suggest that the relation between inflation and economic growth is nonlinear[1]. In addition, recent econometric results exhibit threshold effects of inflation on growth, confirming the nonlinear relation between monetary policy and long-run growth[2].

The goal of this paper is to provide a theoretical model that reproduces these threshold effects in both monetary and fiscal policies, as the result of optimal choice of government finance strategies. Few papers deal with the optimal policy mix in terms of long-run growth and welfare. One very interesting paper is Palivos & Yip (1995), who develop an endogenous growth cash-in-advance (hereafter CIA) model. In their model, the ratio of public spending to GDP is exogenous, and both flat-rate taxes and seigniorage weaken long-run growth. The best strategy for growth maximization is to finance public spending by seigniorage only. Concerning welfare, results essentially depend on the share of private investment that is subjected to the CIA constraint: if this share is small enough, government should use a mix of taxes and seigniorage, while it should use seigniorage only if this share is large. Therefore, if a large part of private investment is money-constrained, maximizing long-run economic growth and/or welfare requires seigniorage-only financing, with zero taxes.

However, these findings are somewhat questionable. First, Palivos & Yip (1995) do not exhibit any threshold effects of monetary or fiscal policy, when empirical evidence suggests such effects. Second, the fact that maximizing growth (and a large range of maximizing welfare) strategies impose seigniorage-only financing is rather bothering, as developed countries use

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[1] For example, Thirlwall & Barton (1971) identify positive effects of below-8%-inflation on growth and negative ones above 10%. Gylfasson (1991) associates high growth countries to inferior-to-5%-inflation and low growth countries to inflation above 20%. Sarrel (1996) finds a breakpoint in the inflation-growth relation at about 8%. More recently, Adam & Bevan (2005) underline threshold effects in both monetary and fiscal policies, a result consistent with our model.

[2] Among these studies, see Bullard & Keating (1995), Boyd, Levine & Smith, (1997), Bruno & Easterly (1998), Kim & Willett (2000) and Adam & Bevan (2005).

little or no seigniorage, while this resource still remains an essential way of government finance for low-developed countries[3]. Thus, it would be interesting to have a model that gives account of this difference and in which the level of seigniorage is optimally chosen by authorities, with respect to structural economic indicators. Third, the optimal inflation rates resulting from seigniorage-only strategies are "stratospheric" for admissible parameter calibrations. Even if some countries temporarily launch into inflationary finance, one may doubt that this is an optimal way to maximize long-run economic growth.

In this paper, we try to offer some answers to those questions, in the spirit of Palivos & Yip (1995). To deal with threshold effects of monetary and fiscal policies, we consider a model with productive public spending (in line with Barro, 1990). Similarly to Palivos & Yip (1995), we first introduce money demand *via* a CIA constraint, but we consider that only part of money supply ("high-powered money") can be used for government finance. Effectively, in developed countries, most of seigniorage is retrieved by private banks, and only seigniorage on the monetary base is appropriable by the central bank. Then, we interpret the "money multiplier" (namely the ratio of the total stock of money to high powered money) as an indicator of financial development.

Using productive public spending, our model produces threshold effect between fiscal policy and economic growth, as in Barro (1990). If the CIA constraint affects consumption only, no threshold effect characterizes the link between monetary policy and long-run economic growth, since seigniorage unambiguously increases growth. This positive effect comes from the fact that seigniorage is similar to a flat tax rate on consumption. With a CIA constraint on all transactions, on the contrary, an inflation tax distorts private capital accumulation, and the model exhibits threshold effects between seigniorage and economic growth, in line with non-linearities described by the econometric literature on the topic. In this case, we obtain threshold effects in both fiscal and monetary policies, which induce seigniorage and tax ceilings and allow precisely comparing the long-run effects of these two ways of government finance. We show that the maximizing-growth and/or welfare strategies are always corner solutions, with only one active instrument, but depending on the "financial development" indicator. Countries with large financial sector must choose tax-only strategies, while less financially developed countries must choose seigniorage-only strategies. These results are valid for both growth and welfare maximization, although the associated strategies do not perfectly correspond.

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[3] In a cross-countries study, Cukierman et *al.* (1992) find that seigniorage represents between 1.7% and 28% of total government revenues. Basu (2001) provides some interesting references on the hypothesis that seigniorage could be an important source of public infrastructure finance.

However, the absence of an interior solution for growth or welfare maximization is a quite unrealistic feature. Consequently, we slightly modify the model by introducing a more general form of money demand based on “transaction costs”. In this more general framework, money demand becomes interest elastic and the model is able to produce interior solutions for long-run growth or welfare maximization, while preserving the key role of financial development indicators in assessing growth and/or welfare maximization strategies.

Section one presents the model, section two analyzes a CIA constraint on consumption only, while section three develops the CIA constraint on all transactions. In section four, we introduce a more general money demand and highlight the existence of an interior solution, and section five provides some empirical evidence.

## 1. The model

Our model depicts a closed economy with three infinitely-lived agents: a producer-consumer representative household, government and monetary authorities. The representative household maximizes the present value of the discounted sum of instantaneous utility based on consumption ($c_t > 0$), with $\rho > 0$ the subjective discount rate:

$$U = \int_0^{\infty} u(c_t)\exp(-\rho t)dt \tag{1}$$

To obtain an endogenous growth path, we assume an isoelastic instantaneous utility function:

$$u(c_t) = \begin{cases} \frac{S}{S-1}\left((c_t)^{\frac{S-1}{S}} - 1\right), & for \;\; S \neq 1 \\ Log(c_t), & for \;\; S = 1 \end{cases} \tag{2}$$

For intertemporal utility $U$ to be bounded, we also must ensure that $\gamma_c(S-1)/S < \rho$, with $\gamma_x$ the long-run growth rate of variable $x$.[4] Output $(y_t)$ is created with private capital $(k_t)$ and productive public expenditures $(g_t)$, with $1/2 < \alpha < 1$ output to private capital elasticity:

$$y_t = k_t^{\alpha} g_t^{1-\alpha} \tag{3}$$

Population is normalized to unity, so that all variables are per capita variables. Public expenditures enter as a flow in the production function and no congestion is present, so that this

[4] This condition corresponds to a no-Ponzi game constraint, $\gamma_c < r$.

function is comparable to Barro (1990).[5] The representative agent budget constraint is, in real variables (a dot over a variable denotes its time derivative):

$$\dot{k}_t + \dot{m}_t = (1-\tau) y_t - c_t - \delta k_t - \pi_t m_t + x_t \tag{4}$$

We assume a flat tax rate on output $(\tau)$. Households use their (net of tax) income $((1-\tau) y_t)$ to consume $(c_t)$, invest $(z_t = \dot{k}_t + \delta k_t)$, with $\delta$ the private capital depreciation rate, and pay taxes. The stock of real balances is $m_t = M_t / P_t$, with $M_t$ the stock of nominal money and $P_t$ the price level. $\pi_t = \dot{P}_t / P_t$ is the inflation rate, so the real stock of money depreciation per unit of time is $\pi_t m_t$. Finally, households receive a lump-sum transfer $x_t$ from banks.

To motivate a money demand, we introduce a *cash-in-advance* constraint on consumption and/or productive expenditures (*i.e.* private investment plus public spending[6]):

$$c_t + \phi(z_t + g_t) = m_t \tag{5}$$

Parameter $\phi$ allows studying two cases: *i)* $\phi = 0$ for a CIA constraint on consumption only, and *ii)* $\phi = 1$ for a global CIA constraint on all transactions.[7]

Let us look more precisely at the monetary side of the model. The nominal stock of money $(M_t)$ is supplied by the banking system (since the CIA constraint motivates a transaction demand for money, we may consider that $M$ includes all means of payment: cash balances and deposit accounts). Monetary authorities set an exogenous nominal stock of high-powered money $(B_t)$, and we suppose that money supply is linked to high-powered money by a standard "money multiplier" $(1/h > 1)$: $M_t = (1/h) B_t$. Coefficient $h$ depends on the ratio of banknotes to the nominal stock of money and on bank reserve requirements, which we do not model explicitly.[8] Equilibrium on the money market will determine the price level $P_t = M_t / m_t = B_t / h m_t$. Thus, in our model, "money" used in transactions must be distinguished

[5] Condition $0 < \alpha < 1$ ensures the existence of a competitive equilibrium, since $g_t$ is exogenous for households and the production function exhibits decreasing returns to scale. On the contrary, $g_t$ will be endogenously obtained in equilibrium, and production will exhibit constant returns, a necessary condition for a constant growth path in the long run. Condition $\alpha > 1/2$ allows obtaining positive ceiling for the money growth rate (see *section 3* below).

[6] *Appendix 4* shows that with a CIA constraint on consumption and private investment only (a case first studied by Stockman, 1981, in an exogenous growth setup) does not qualitatively change the model. Including public spending in the CIA yields a simple money demand, $m = y$ in equilibrium, which simplifies calculations. We assume a strictly positive interest rate, so that the CIA constraint holds as equality.

[7] Palivos & Yip (1995) study the $0 \le \phi \le 1$ case, but we cannot obtain simple analytical results for $\phi \in (0,1)$, although simulations show that our main results are qualitatively unchanged (see *Appendix 4*).

[8] A little more structure could be easily introduced in the model, by considering as in Englund & Svensson (1988) or Hartley (1988), two types of goods: "cash goods", to be paid with cash reserves and "check goods" which need bank deposits. The money multiplier $h$ would depend on the relative weight of the two goods in total consumption.

from “high-powered money” which provide seigniorage for government finance. In developed countries, most of money is generated by private banks, and the stock of money which serves for transaction (the one included in the CIA constraint) is much larger than the stock of high-powered money. Therefore, it should be unreasonable to assume that all seigniorage on money is collected by monetary authorities. Coefficient $h$ may be analyzed either as an indicator of “financial (or banking) under-development” or as an indicator of “financial repression” (since there is a strong positive association between financial repression and low financial development, as emphasized by Haslag & Koo, 1999).

We are interested in monetary policies that set an exogenous stock of high-powered money growth rate $\left(\dot{M}_t / M_t = \dot{B}_t / B_t = \omega\right)$. Monetary authorities receive seigniorage on high-powered money (in real variables: $\omega B_t / P_t = h\omega m_t$) and transfer it to government, which builds its resources not only from taxes on output $\left(\tau y_t\right)$ but also from seigniorage:[9]

$$g_t = \tau y_t + h\omega m_t \tag{6}$$

Relation (6) departs from the Barro (1990) budget constraint $g_t = \tau y_t$, since seigniorage can be used for government finance, as in Palivos & Yip (1995). Finally, to close the model, we suppose that the banking system transfers to households the (lump-sum) real value of “private seigniorage”, $x_t = (1-h)\,\omega m_t$.

*Steady-state equilibrium*

The representative agent maximizes (1) subject to (2)-(3)-(4)-(5), $k_0$ given and a standard transversality condition (see *Appendix 1* for details). Using intensive variables: $c_k = c/k$, $m_k = m/k$ and $g_k = g/k$, we find the steady-state endogenous growth solution by imposing $\dot{c}_k = \dot{m}_k = \dot{g}_k = 0$, which implies constant values of $c_k$, $m_k$ and $g_k$, so that initial variables ($c$, $k$, $g$, $m$) grow at the same constant rate $\gamma$. *Appendix 1* shows that steady-state solution can be depicted by two relations between $\gamma$ and $g_k$:

$$\gamma = S\left[\frac{\alpha(1-\tau)\,g_k^{1-\alpha}}{1+\phi\left(\omega + s(\gamma)\right)} - \delta - \rho\right], \text{ with } s(\gamma) \equiv \rho - \gamma(S-1)/S \tag{7}$$

$$g_k = \frac{(\tau + h\omega)g_k^{1-\alpha} - h\omega(1-\phi)(\gamma+\delta)}{1 + h\omega(1-\phi)} \tag{8}$$

[9] It should clearly appear that we introduce coefficient $h$ to take account of the fact that only a part of the “inflation tax” perceived by the banking system may be used to finance government spending (that is, the inflation tax on “money base”: banknotes and bank reserves).

Equation (7) is simply the Keynes-Ramsey relation in steady state: $\dot{c}/c = \gamma = S(r-\rho)$, with $r$ the real interest rate. If the CIA constraint affects only consumption $(\phi = 0)$, the real interest rate correspond to the net return of private investment: $r = (1-\tau) f_k(k) - \delta = \alpha(1-\tau) g_k^{1-\alpha} - \delta$. If investment is money-constrained $(\phi > 0)$, the return of private investment must be deflated by the transaction cost on new capital goods $(1+\phi R)$, where $R$ is the nominal interest rate and $R = \omega + r - \dot{m}/m = \omega + r - \gamma = \omega + s(\gamma)$ in steady-state.

Equation (8) comes from the government constraint (6), with the real balances stock from the CIA constraint (5): $m_k = c_k + \phi\left(g_k^{1-\alpha} - c_k\right)$, with $c_k = g_k^{1-\alpha} - g_k - \delta - \gamma$ in equilibrium.

Steady-state solution corresponds to the intersection of (7) and (8), which simultaneously provide the long-run growth rate $(\gamma)$ and the long-run endogenous public spending to private capital ratio $(g_k)$. *Appendix 1* shows that there are only jump variables in the dynamic system governing the economy, thus, whatever the $\phi$ value, the model lacks any transitional dynamics and variables instantly jump to theirs steady-state values (see *Appendix 5* for a formal analysis).

We distinguish two cases, according to the form of the CIA constraint: a constraint on consumption only $(\phi = 0)$, and a constraint on all transactions $(\phi = 1)$. Thus, our model involves two differences compared to Palivos & Yip (1995). First, we do not consider the $0 < \phi < 1$ case (however, *Appendix 4* extends our main results to this case), because we wish to focus the analysis on the role of the financial development indicator $(h)$, without making the model complex. Second, Palivos & Yip (1995) consider exogenous unproductive public spending which represent a fixed percentage of GDP, while we are interested in endogenous productive spending, as in Barro (1990). One can obtain Palivos & Yip results in our model by considering an exogenous stream of public spending $\bar{g}_k$.

## 2. A CIA constraint on consumption only

With a CIA constraint on consumption only $(\phi = 0)$, we obtain, from the total differential of system (7)-(8):

$$\frac{d\gamma}{d\omega} = \frac{\alpha h S(1-\alpha)(1-\tau)c_k}{\left[\omega h \alpha S(1-\tau) - (\tau + h\omega)\right](1-\alpha) + (1+h\omega) g_k^{\alpha}} \tag{9a}$$

with $d\gamma/d\omega > 0$ on the sufficient condition $S < \bar{S} \equiv (\tau + h\omega)/[\alpha h \omega(1-\tau)]$, which we suppose.

Any increase in the money growth rate unambiguously raises long-run economic growth. As a corollary of this result, long-run growth is higher than the Barro (1990) solution without seigniorage. Effectively, without seigniorage $(h=0)$, productive public spending is $\hat{g}_k = \tau^{1/\alpha}$ (the Barro solution). With seigniorage $(h>0)$, it becomes, from (8) with $\phi = 0$: $g_k^* = \left[\tau + h\omega m_k^* / (g_k^*)^{1-\alpha}\right]^{1/\alpha} > \hat{g}_k$, for positive $m_k^*$, $g_k^*$ and $\omega$ (the star denotes steady-state values). Therefore, using seigniorage provides more resources for productive government expenditures that are long-run growth enhancing. Furthermore, the inflation-tax has no effect on capital accumulation, since the CIA constraint affects consumption only. Consequently, seigniorage is similar to a flat-rate tax on consumption, therefore *always* growth enhancing.[10]

To assess the effect of taxes on long-run growth, we proceed by a similar way. From the total differential of system (7)-(8), we obtain:

$$\frac{d\gamma}{d\tau} = \frac{-\alpha S g_k^{1-\alpha}(1+h\omega)\left[g_k^{\alpha} - (1-\alpha)\right]}{(1+h\omega)g_k^{\alpha} + (1-\alpha)\left[h\omega\alpha S(1-\tau) - \tau - h\omega\right]} \tag{9b}$$

Thus, the capital ratio $g_k^{\alpha} = (1-\alpha)$ corresponding to the long-run growth maximizing-tax rate is the same as Barro (1990). From the government budget constraint (8) we extract the maximizing-growth tax-rate $(\tau^M)$:

$$\tau^M = g_k^{\alpha} - h\omega\left[g_k^{1-\alpha} - g_k - \gamma - \delta\right]g_k^{\alpha-1} = 1-\alpha - h\omega(c/y)^* \leq 1-\alpha \tag{10}$$

If $h\omega = 0$, the maximizing-growth tax-rate is: $\tau^B = 1-\alpha$, as in Barro (1990). If governments use seigniorage to finance public expenditures, on the contrary, the maximizing-growth tax-rate is lower: $\tau^M < \tau^B$. Effectively, since the capital ratio corresponding to the long-run growth maximizing-tax rate $\left(g_k^{\alpha} = (1-\alpha)\right)$ is the same as Barro (1990), with money financing the growth-maximizing tax rate will be smaller than without. The term $h\omega(c/y)^*$ in equation (10) represents the additional charge provided by money financing.

To resume, with $\phi = 0$, there is no optimal mix of government finance from a long-run growth perspective. Since seigniorage is always growth-enhancing and there is a ceiling for taxes, growth-oriented governments[11] should use large seigniorage rates, or even money-only financing strategies. However, these findings are questionable, since an important number of countries collect theirs resources mainly from taxes. Furthermore, empirical evidence shows that

[10] This result meets numerous works in endogenous growth models without leisure, showing the benefits of consumption taxes rather than taxes on output that penalize capital accumulation (see, *e.g.* Turnovsky, 1996).
[11] We study inflation and welfare in the following section.

inflationary finance gives rise to a “Laffer curve” of seigniorage, thus revenues that can be built from an inflation tax are limited. Finally, many applied studies report threshold effects in the relation between money growth (or inflation) and economic growth. To reproduce these evidences, we introduce a generalized CIA constraint.

## 3. A generalized cash-in-advance constraint

To obtain threshold effects in monetary policy, we introduce a CIA constraint on private investment. We also impose the CIA constraint on public spending, since the model with a generalized CIA constraint is simpler than a model with a constraint on consumption and private investment only, without any qualitative change (see *Appendix 4*). If $\phi = 1$, equation (8) does not depend on $\gamma$, and we directly obtain the public spending to capital ratio:

$$g_k = \left(\tau + h\omega\right)^{1/\alpha} \tag{11}$$

Reintroducing this value in (7) yields an implicit relation for long-run economic growth $\left(\gamma^*\right)$:

$$\left(\gamma^* / S + \delta + \rho\right)\left[1 + \omega + s\left(\gamma^*\right)\right] = \alpha(1-\tau)(\tau + h\omega)^{(1-\alpha)/\alpha} \tag{12}$$

Relation (12) is a second degree polynomial in $\gamma^*$, with generally only one positive solution.[12] We first outline analytical results for $S = 1$, then we present numerical simulations for the general case $S \neq 1$, showing that our analytical results are qualitatively unchanged.

In the **particular case** $S = 1$, equation (12) reduces to:

$$\gamma^* = \gamma^*\left(\tau, \omega\right) = \frac{\alpha\left(1-\tau\right)\left(\tau + h\omega\right)^{\frac{1-\alpha}{\alpha}}}{1+\omega+\rho} - \delta - \rho \tag{13}$$

Relation (13) clearly exhibits a threshold between money and long-run economic growth. Effectively, any increase in seigniorage is devoted to productive public expenditures that are growth-enhancing (numerator of (13)), but such an increase simultaneously raises the financing cost of private investment, which is harmful to long-run growth (denominator of (13)). The trade-off between these two effects, illustrating that productive public spending crowd out

[12] Relation (12) can be written $\left(\gamma^*\right)^2 + B\gamma^* + C = 0$, where $B \equiv S\left[(1+\omega+\rho)/(1-S) + \delta + \rho\right]$, $C \equiv AS^2/(S-1)$ and $A = \alpha(1-\tau)(\tau + h\omega)^{(1-\alpha)/\alpha} - (\delta+\rho)(1+\omega+\rho)$. This polynomial has two roots $\gamma_1 = \left(-B - \sqrt{B^2 - 4C}\right)/2$ and $\gamma_2 = \left(-B + \sqrt{B^2 - 4C}\right)/2$. As we shall see, a necessary condition for the growth rate to be positive if $S = 1$ is: $A > 0$, that we suppose. If $S < 1$ then $C < 0$, and $\gamma_2$ is the only positive solution. If $S > 1$ then $C > 0$ and a sufficient condition for $\gamma_2$ to be the only positive solution is $S < 1 + (1+\omega+\rho)/(\delta+\rho)$, that we also suppose.

private investment, results in a ceiling for $\omega$, say $\bar{\omega}$. We compute this ceiling using the first order condition for the maximization of (13):

$$\frac{\partial\gamma^*(\tau,\omega)}{\partial\omega}=0\Rightarrow\bar{\omega}=\frac{(1-\alpha)h(1+\rho)-\alpha\tau}{h(2\alpha-1)} \quad (14a)$$

Notice that this "Laffer curve" of seigniorage reproduces numerous empirical evidences describing a non-linear relation between seigniorage (or inflation)[13] and growth (see *footnote 1*).

As in Barro (1990), our model also exhibits an inverted-U curve between fiscal policy and long-run growth. The growth-maximizing tax-rate $(\bar{\tau})$ is independent of $S$ and can easily be computed from either (12) or (13):

$$\frac{\partial\gamma^*(\tau,\omega)}{\partial\tau}=0\Rightarrow\bar{\tau}=1-\alpha-\alpha h\omega<1-\alpha \quad (15)$$

As in the previous section, this value is lower than the Barro's one, if $h\omega>0$. With a CIA constraint on investment expenditures, contrary to the previous section, the public capital ratio corresponding to the tax rate that maximizes economic growth is $g_k^\alpha=(1-\alpha)(1+h\omega)$, and differs from its value in Barro (1990). Effectively, to maximize economic growth in (7), the tax rate must obey to the following relation:

$$-g_k^{1-\alpha}+(1-\alpha)(1-\tau)g_k^{-\alpha}\frac{dg_k}{d\tau}=0 \quad (16a)$$

The first term represents the marginal cost of taxation, and the second term its marginal benefit (the income tax provides resources for productive public spending). In the government budget constraint (8) with $\phi=1$, we find: $dg_k/d\tau=g_k^{1-\alpha}/\alpha$, thus (16a) becomes:

$$-\alpha+(1-\alpha)(1-\tau)g_k^{-\alpha}=0 \quad (16b)$$

Without seigniorage, as in Barro (1990), the government budget constraint is simply: $g_k^\alpha=\tau$, and we find the maximizing-growth tax-rate: $\tau^B=1-\alpha$. With seigniorage, on the contrary: $g_k^\alpha=\tau+h\omega$, and the marginal return of taxes is lower than without money in relation (16b), which explains both why the public capital ratio $g_k$ is higher than the Barro's one and why the growth-maximizing tax rate must be lower than in a model without seigniorage.

However, observe that relations (14a) and (15) are the result of growth maximization by only one instrument, keeping the second given. If we consider simultaneous changes in the two instruments in order to maximize economic growth, we must verify not only first order conditions but also some second order conditions.

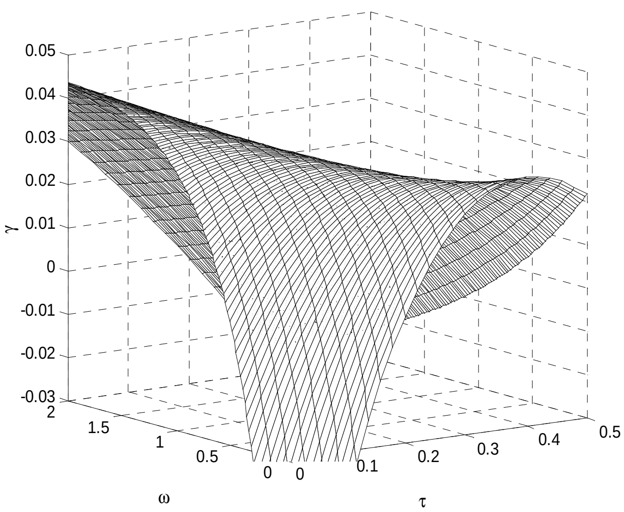
[13] Generally, the long-run inflation rate ($\pi=\omega-\gamma$) positively depends on seigniorage.

For an interior solution $(\bar{\bar{\tau}}, \bar{\bar{\omega}})$ to exist, first order conditions must be simultaneously respected. From (14a) and (15), we find immediately one (and only one) maximum candidate:

$$\bar{\bar{\tau}} = \frac{(1-2\alpha)+\alpha h(1+\rho)}{1-\alpha} \qquad \text{and} \qquad \bar{\bar{\omega}} = \frac{\alpha - h(1+\rho)}{h(1-\alpha)} \tag{17}$$

Nevertheless, the $(\bar{\bar{\tau}}, \bar{\bar{\omega}})$ couple never is an optimum, since second order conditions are not fulfilled (see *Appendix 2*). On the contrary, the $\gamma^* = \gamma^*(\tau, \omega)$ curve forms a "pass" at this point as shows *Figure 1*.

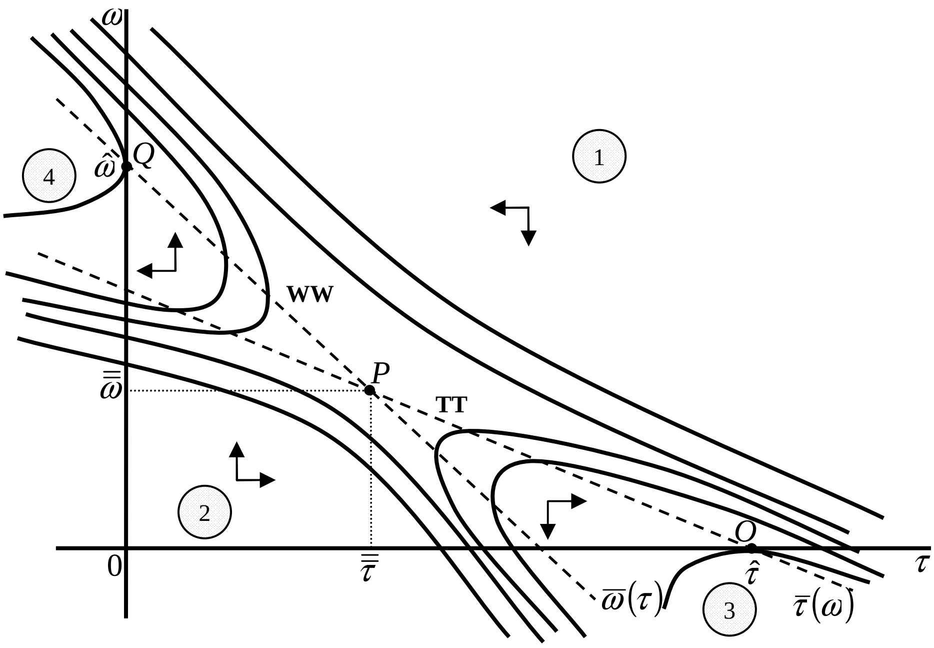
*Figure 1 – Long-run economic growth rate as a function of* $\tau$ *and* $\omega$[14]




In the absence of interior solution, let us focus on corner solutions. To derive some graphical results, *Figure 2* depicts economic growth contour lines in the $(\tau, \omega)$ plane, namely combinations of the tax and money growth rates leading to a similar long-run growth rate.

[14] All along the paper we use, unless otherwise indicated, simulation values: $S=1$, $\alpha=0.6$, $\rho=0.1$, $\delta=0.05$ and $h=0.5$. We confirm that results are strongly robust to changes in parameters.

*Figure 2 – Contour lines of economic growth as a function of $\tau$ and $\omega$*




In *Figure 2*, arrows indicate that long-run economic growth increases.[15] For a given money growth rate, say $\omega_0$, any increase in the tax rate raises economic growth, up to the value $\bar{\tau}(\omega_0)$ (the TT line on *Figure 2*). Beyond this point, economic growth is negatively linked to the tax rate. Thus, iso-growth curves present a horizontal tangent on the TT line. Similarly, for a given rate of the income tax-rate, say $\tau_0$, any increase in seigniorage raises economic growth, up to the value $\bar{\omega}(\tau_0)$ (the WW line on *Figure 2*). Beyond this point, money and economic growth are negatively related. Hence, iso-growth curves present a vertical tangent on the WW line. Thus, we can split the $(\tau, \omega)$ plane in four regions. In *Region 1* higher tax and/or money growth rates lower long-run economic growth. In *Region 2* on the contrary, they induce higher growth rates. In these two areas, the instruments are substitutes: the same economic growth rate can be achieved by increasing (respectively decreasing) taxes and decreasing (respectively increasing) money creation. On the contrary, in *Regions 3* and *4* the two instruments are complementary: to reach the same economic growth rate, government must increase simultaneously income-tax and seigniorage, since economic growth depends positively on taxes and negatively on seigniorage in *Region 3*, with reversed properties in *Region 4*.

[15] Observe that the slope of $\bar{\omega}(\tau)$ is higher than the slope of $\bar{\tau}(\omega)$. *Figure 2* is established for sufficiently high values of $h$, namely $h > (2\alpha - 1)/[\alpha(1+\rho)]$, otherwise the two curves intersect for negative values of $\omega$, however without qualitative change in our findings.

As one can easily remark in *Figure 2*, $P$ point is not an optimum. Two local maxima may eventually emerge in *Regions 3* and *4* for negative values of the instruments, which we exclude. Thus, we look for corner solutions subject to non-negativity constraints $\tau \geq 0$ and $\omega \geq 0$. Such corner solutions provide two growth-maximizing candidates, $(0, \hat{\omega})$ and $(\hat{\tau}, 0)$, namely the tangency points between the highest contour-line and the axis $\tau = 0$ or $\omega = 0$. Choosing between these two maxima is a matter of parameter values. In the first case ($Q$ point in *Figure 2*) we obtain $\tau = 0$ and $\omega = \hat{\omega} = (1-\alpha)(1+\rho)/(2\alpha - 1)$, and the related growth is:

$$\gamma^*(0, \hat{\omega}) = \frac{2\alpha - 1}{1+\rho}\left(\frac{h(1-\alpha)(1+\rho)}{2\alpha - 1}\right)^{(1-\alpha)/\alpha} - \delta - \rho \tag{18a}$$

In the second case ($O$ point in *Figure 2*), $\omega = 0$ and $\tau = \hat{\tau} = 1 - \alpha$, and the long-run growth is:

$$\gamma^*(\hat{\tau}, 0) = \frac{\alpha^2(1-\alpha)^{(1-\alpha)/\alpha}}{1+\rho} - \delta - \rho \tag{18b}$$

Either the $(0, \hat{\omega})$ point or the $(\hat{\tau}, 0)$ point may be the global maximum for $\gamma^* = \gamma^*(\tau, \omega)$, since: $\gamma^*(\hat{\tau}, 0) >< \gamma^*(0, \hat{\omega}) \Leftrightarrow h <> \bar{h}$, with $\bar{h} = \frac{1}{1+\rho}\left[\alpha^{2\alpha}(2\alpha - 1)^{1-2\alpha}\right]^{1/(1-\alpha)}$. Therefore, if the money multiplier is large $(h < \bar{h})$, matching an important seigniorage flight from the central bank to the private banking sector, the optimal way of finance for a growth-oriented government is an "income-tax only strategy", while a "money only strategy" in countries with less developed financial sectors $(h > \bar{h})$. This result generalizes Palivos & Yip (1995) model, in which the maximizing-growth strategy always implies a money-only strategy, because $h = 1$.

In the **general case** $S \neq 1$, the tax-rate ceiling is unchanged, as we have seen. The total differential of (12) with $d\gamma / d\omega = 0$, defines an implicit relation for the money growth ceiling:

$$\bar{\omega} = \frac{(1-\alpha)h\left[1 + s(\gamma^*)\right] - \alpha\tau}{(2\alpha - 1)h} \tag{14b}$$

where $\gamma^* = \gamma^*(\bar{\omega}, \tau)$ is defined by (12). Notice that (14a) and (14b) are identical if $S = 1$.

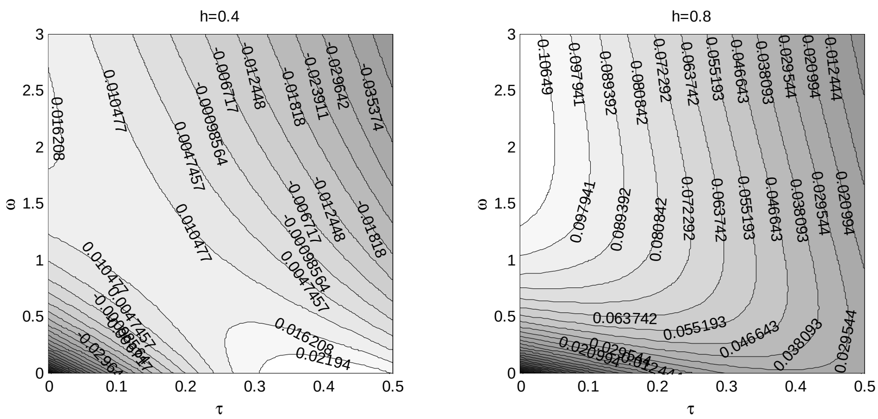
*Table 1* reports some simulation values for the three particular equilibria $O$, $P$ and $Q$, for different values of $S$ and $h$. In each case, the values of income-tax and seigniorage rates are computed using relations (12) and (14b). $O$ point is such that $\omega = 0$ and $\tau = 1 - \alpha$. Money growth rates associated to $P$ and $Q$ points respectively are obtained by replacing in these relations the tax rate with its relevant value ($\tau = 1 - \alpha - \alpha h \omega$, respectively $\tau = 0$).

*Table 1 – Maximizing-growth candidates*

| | | $S=0.5$ | | $S=1$ | | $S=1.5$ | |
|---|---|---|---|---|---|---|---|
| | | $h=0.4$ | $h=0.8$ | $h=0.4$ | $h=0.8$ | $h=0.4$ | $h=0.8$ |
| $O$ point | $\hat{\tau}=1-\alpha$ | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| | $\omega=0$ | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\gamma$ | 1.28% | 1.28% | 2.77% | 2.77% | 4.52% | 4.52% |
| $P$ point | $\tau=\overline{\overline{\tau}}$ | 0.164 | 0.80 | 0.16 | 0.82 | 0.156 | 0.85 |
| | $\omega=\overline{\overline{\omega}}$ | 0.98 | -0.84 | 1 | -0.875 | 1.02 | -0.93 |
| | $\gamma$ | 0.63% | -1.32% | 1.31% | -3.32% | 2.04% | -6.79% |
| $Q$ point | $\tau=0$ | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\omega=\hat{\omega}$ | 2.22 | 2.31 | 2.2 | 2.2 | 2.18 | 2.08 |
| | $\gamma$ | 0.83% | 5.54% | 1.7% | 11.5% | 2.61% | 18.01% |

These findings show that the global maximum of $\gamma^*=\gamma^*(\tau,\omega)$ is still either the $(0,\hat{\omega})$ point or the $(\hat{\tau},0)$ point, depending on parameters, and especially on the value of the money multiplier $(h)$. In *Table 1*, if $h=0.4$, the optimal government finance strategy is an income-tax only strategy, while it swings to a seigniorage-only strategy if $h=0.8$, confirming our previous results for $S=1$. *Figure 3* illustrates these two typical situations.

*Figure 3 – Contour lines of economic growth as a function of $\tau$ and $\omega$*




These results are robust to changes in parameters: in all simulations we performed, one can find long-run growth maximizing $h$ values leading to opposite structure for government finance.

*Welfare analysis*

We are exclusively interested in "second best" welfare issues, in which a benevolent government chooses an income tax and a seigniorage rate that maximize households' welfare in a decentralized economy. This solution is different from the "first best" solution, which requires *i)* lump-sum taxation, *ii)* eliminating the opportunity cost of holding money, by adopting a

Friedman-like rule[16]: $\omega = -\rho + (S-1)\gamma / S$ . Under these two set of policies, the rate of return of investment is not distorted by the type of government finance, and the first-best economic growth rate is: $\gamma^{opt} = S\left[\alpha(1-\alpha)^{(1-\alpha)/\alpha} - (\delta+\rho)\right]$. This first-best solution corresponds to what a central planner would obtain by maximizing households' utility (1), under the aggregate budget constraint (4+6), money market equilibrium and the definition of the financial transfer $(x)$. Indeed, for such a central planner, the relevant budget constraint is the *IS* equilibrium and the CIA constraint is ineffective if the nominal interest rate comes to zero. Thus, the central planner directly chooses the ratio of public spending $g_k = (1-\alpha)^{1/\alpha}$, as in Barro (1990), and the first-best economic growth rate or intertemporal welfare are independent of the policy mix. On the contrary, under a second-best strategy, households maximize utility in a decentralized economy and government chooses *ex-post* the rates of taxes and seigniorage that maximize welfare.

To assess intertemporal welfare, we perform some stability analysis. *Appendix 5* shows the absence of transitional dynamics in our model, similar to Barro (1990) or Palivos & Yip (1995). Thus, we can express intertemporal welfare in a very simple form, with: $c_0 = k_0\left[g_{k0}^{1-\alpha} - g_{k0} - \delta - \gamma^*\right]$, $g_{k0} = (\tau + h\omega)^{1/\alpha}$ and $\gamma^*$ defined in (12):

$$U = \begin{cases} \dfrac{S}{S-1}(c_0)^{\frac{S-1}{S}}\left[\rho - \gamma^*\left(\dfrac{S-1}{S}\right)\right]^{-1} + \dfrac{S}{\rho(1-S)} & if \quad S \neq 1 \\ \left[Log(c_0) + \gamma^*/\rho\right]/\rho & if \quad S = 1 \end{cases} \tag{19}$$

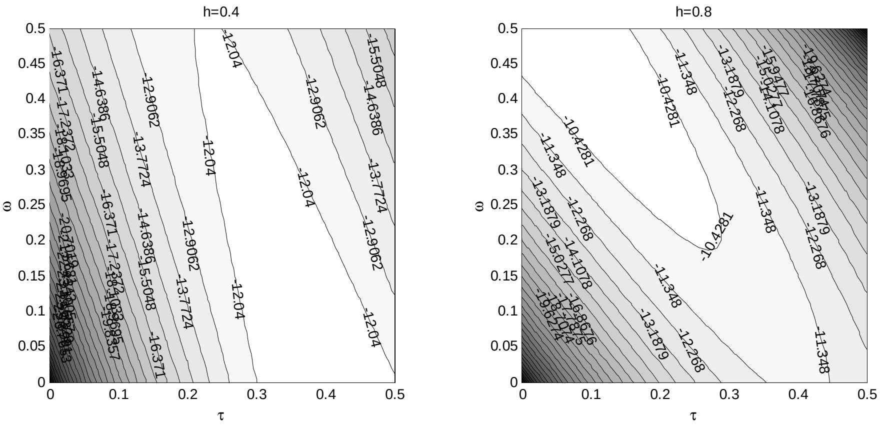
It is rather difficult to derive analytic optimal values of taxes and seigniorage, but simulations clearly show that welfare-maximizing optimal solutions are still corner solutions[17]. Moreover, as in *Figure 4*, the optimal couple switches from $(\tilde{\tau}, 0)$ if financial development is high ($h = 0.4$ in *Figure 4)*, to $(0, \tilde{\omega})$ in low financial development countries ($h = 0.8$ in *Figure 4)*.

---

[16] If $S = 1$, we find the Friedman (1969) rule strictly speaking $(\omega = -\rho)$. But, in a context of positive long-run growth, the nominal interest rate does not come to zero when $\omega = -\rho$, unless $S = 1$.

[17] Palivos & Yip (1995) obtain an interior solution for welfare maximization if $0 < \phi < 1$, a result that our model also produces. In the next section we propose an interior solution *for economic growth and welfare* even for $\phi = 1$.

*Figure 4 – Contour lines of welfare as a function of $\tau$ and $\omega$*




The tax rate that maximizes intertemporal welfare when $\omega = 0$ is the one that maximizes economic growth $\tilde{\tau} = \hat{\tau} = 1-\alpha$, as in Barro (1990). On the contrary, the seigniorage rate that maximizes intertemporal welfare $(\tilde{\omega})$ when $\tau = 0$ is different from the one that maximizes economic growth $(\hat{\omega})$. Effectively, if $S = 1$ for example:

$$\frac{dU}{d\omega} = \frac{1}{\rho}\left[\frac{1}{c_0}\frac{dc_0}{d\omega} + \frac{1}{\rho}\frac{d\gamma^*}{d\omega}\right] = \frac{1}{\rho}\left[\frac{h\left[(1-\alpha)-h\omega\right]}{\alpha c_{k0}}(h\omega)^{1-2\alpha/\alpha} + \left(\frac{1}{\rho}-\frac{1}{c_{k0}}\right)\frac{d\gamma^*}{d\omega}\right] \qquad (20)$$

Putting $\omega = \hat{\omega} = (1-\alpha)(1+\rho)/(2\alpha-1)$ (thus $d\gamma^* / d\omega = 0$) in (19), remark that welfare is not maximized, since $\left.\frac{dU}{d\omega}\right|_{\omega=\hat{\omega}} = \left[(2\alpha-1)-h(1+\rho)\right]\frac{h(1-\alpha)(h\hat{\omega})^{1-2\alpha/\alpha}}{\alpha(2\alpha-1)\rho c_{k0}} >< 0 \;\; if \;\; h <> \frac{2\alpha-1}{1+\rho}$.

Thus, the money growth rate maximizing intertemporal welfare may be higher (if $h$ is small) or lower (if $h$ is high) than the one that maximizes economic growth. Consequently, there may be a wedge between maximizing economic growth and maximizing welfare. Effectively, the ceiling value of $h$ which corresponds to a switch in growth-maximizing strategies $(\bar{h})$ differs from the ceiling value corresponding to a switch in welfare-maximizing strategies (say, $\bar{\bar{h}}$). Simulations show in particular that $\bar{h} < \bar{\bar{h}}$. Thus, if $h < \bar{h}$, maximizing growth and welfare strategies require using taxes only, and if $h > \bar{\bar{h}}$, these strategies involve seigniorage only. But for intermediate values of financial development $\bar{h} < h < \bar{\bar{h}}$, maximizing economic growth requires a seigniorage-only corner solution with no taxes, while maximizing intertemporal welfare requires the opposite corner solution with taxes-only and no seigniorage. In our simulations, these undesirable feature arises for $h$ between $\bar{h} = 0.44$ and $\bar{\bar{h}} = 0.52$.

Beyond the existence of possible switches of corner solutions for the maximization of welfare relative to economic growth, “bang-bang” solutions that our model exhibits, depending on the value of the money multiplier, raise two associated questions. First, most countries adopt interior solutions for government finance, with both seigniorage and taxes. Second, corner solutions with seigniorage only conduct to incredibly high money growth (and inflation) rates, even if they may characterize some extreme situations in which governments resort to inflationary finance. Both questions are closely related to the absence in our model of interior solutions for maximizing economic growth and welfare.

An interior solution would eventually arise in a non-cooperative game interpretation of our model, if a government and a central bank simultaneously try to maximize economic growth by choosing their own policy instrument (respectively the income-tax and the seigniorage rate). In such a non-cooperative approach, first order conditions (14a) and (15) would provide “reaction functions” for the government and the central bank respectively, and Nash-equilibrium would be the $P$ point, namely an interior but sub-optimal solution.

Another way to generate interior solutions is to suppose that economic growth is not the only goal of policymakers. If there is a trade-off between inflation and growth, for example, the plausibility of corner solutions without taxation vanishes, because seigniorage directly causes inflation. Starting from $\pi_t = \omega - \gamma_t$, the absence of transitional dynamics implies that the inflation rate is always constant, $\pi^* = \omega - \gamma^*(\omega, \tau)$. Thus, the income-tax threshold on inflation is identical to the one on economic growth $(\overline{\tau} = 1 - \alpha - \alpha h \omega)$, while seigniorage always increases inflation (since $\partial \pi / \partial \omega = 1 - \partial \gamma(\omega, \tau) / \partial \omega > 0$ for admissible parameter values). Consequently, to minimize inflation one must adopt the instruments $\omega = 0$ and $\tau = 1 - \alpha$. Therefore, if governments are interested in both economic growth and inflation, optimal solution can be either a corner solution without seigniorage or an interior solution, depending on the relative weight of the two objectives in the government loss function[18].

Nevertheless, it would be interesting to benefit of a model that directly produces an interior solution for government finance as a result of economic growth maximization only. Let us now slightly modify our model in this perspective.

[18] If $h > \overline{h}$, economic growth is maximized in $Q$ point, while inflation is minimized in $O$ point. Thus, there is a trade-off between inflation and growth, and the optimal solution of this trade-off will generally be an interior solution, which depends on the relative weights of the two objectives in the government loss function.

## 4. A model with transaction costs

In this section, we show how our model can exhibit an interior solution for government finance, if the transaction technology is modified, in line with Feenstra (1986) technology. Suppose that, instead of the CIA constraint, all transactions, (including consumption, investment and government expenditures), are subject to a "transaction cost" $T = T\left(c_t + z_t + g_t, m_t\right)$ and that money provides "liquidity services", by reducing them. For the representative household, the transaction cost affects consumption and investment decisions, since public spending is given, but in equilibrium transaction costs depend on revenue $T = T\left(y_t, m_t\right)$, with usual properties $T_y > 0$, $T_{yy} > 0$, $T_m < 0$, $T_{mm} > 0$ (see Feenstra, 1986). We suppose an isoelastic function:

$$T\left(y_t, m_t\right) = \psi y_t \text{, where: } \psi \equiv \frac{\theta}{\mu}\left(\frac{y_t}{m_t}\right)^{\mu} \tag{21}$$

where $\theta$ is a positive scale-parameter, ensuring "small" transaction costs. In equilibrium, $T\left(.\right)$ express that a fraction $\psi \in \left(0,1\right)$ of output is "lost" in the process of financial intermediation (in line with Pagano, 1993 and Roubini & Sala-i-Martin, 1995). This fraction is inversely related to the ratio of real balances to output ($m_t / y_t$), since money provides "liquidity services". As in the CIA model of the previous section, it is essential that investment expenditures are subject to transaction costs, so monetary policy affects capital accumulation in the long-run.

The model of the previous section is a special case of the technology (21), since (21) gives rise to a CIA constraint when $\mu \to \infty$.[19] The advantage of (21) is that it is more general and yields an interest-elastic money demand (see *Appendix 3*): $m_t = \varphi y_t \left(R_t\right)^{\frac{-1}{1+\mu}} = m\left(\underset{+}{y_t}, \underset{-}{R_t}\right)$, with $\varphi = \theta^{1/(1+\mu)}$. If $\mu = 1$ (as in our simulations below) for example, the demand for real balances joins usual empirical evidences, with an income elasticity of 1 and a nominal interest rate elasticity of $-0.5$.

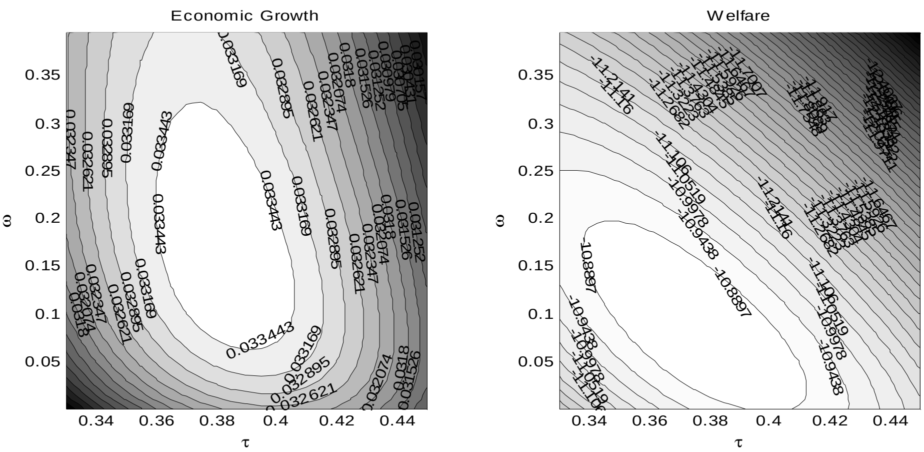
*Appendix 3* solves this transaction costs model for the general case $\left(S \neq 1\right)$. If $S = 1$, we obtain a simple analytic expression for the long-run rate of economic growth:

$$\gamma^* = \frac{\alpha\left(1-\tau\right)\left[\tau + \varphi h \omega\left(\omega + \rho\right)^{-1/(1+\mu)}\right]^{(1-\alpha)/\alpha}}{1 + \varphi\left(\omega + \rho\right)^{\mu/(1+\mu)}} - \delta - \rho \tag{22}$$

[19] Writing $m_t = \left[\frac{\theta y_t}{T\left(y_t, m_t\right)} \frac{1}{\mu}\right]^{1/\mu} y_t$, we use $\lim_{\mu\to\infty}\left(\frac{1}{\mu}\right)^{1/\mu} = \lim_{\mu\to\infty}\left[\exp\left(\frac{1}{\mu} Log\left(\frac{1}{\mu}\right)\right)\right] \to 1$, thus $m_t \to y_t$.

which corresponds to relation (13) if $\mu \to \infty$ ($\varphi = 1$ in the CIA case $\mu \to \infty$). The model can henceforth produce an interior solution for growth and welfare maximization, as in *Figure 5*.

*Figure 5 – Economic growth and welfare contour lines as a function of* $\tau$ *and* $\omega$




*For simulation values:* $S = 1$, $\alpha = 0.6$; $\rho = 0.1$, $\delta = \theta = 0.05$, $h = 0.4$, $\mu = 1$.

Compared to the CIA model of the previous section, this general transaction costs model avoids « bang-bang » solutions. In particular, the critical switch of public finance strategies for the maximization of welfare relative to economic growth disappears. Furthermore, observe that, for parameter values of *Figure 5*, we obtain rather realistic values for the tax rate (about 0.4), the money growth rate (about 0.05) and the related maximized long-run growth rate (about 0.02). Thus, the transaction cost model fits better the data than the CIA model concerning interior solutions, although most of qualitative properties are similar.

To explain why an interior solution arises in the transaction cost model, remark that relations (22) and (13) differ in two ways. First, the elasticity of the cost of financing investment with respect to the money growth rate is lower in the transaction cost model, compared to the CIA case (denominator of equations (13) and (22)). Second, the elasticity of seigniorage resources with respect to the money growth rate is also lower in the transaction cost model than in the CIA case (numerator of equations (13) and (22)), since money negatively depends on the interest rate. The latter property gives rise to the interior solution, while the former is less essential. Effectively, (22) can be rewritten as $\gamma^* = \frac{\alpha(1-\tau)\left[\tau + \varphi h\left(X - Y\right)\right]^{(1-\alpha)/\alpha}}{1+\varphi X} - \delta - \rho$, with $X \equiv (\omega + \rho)^{\mu/(1+\mu)}$ and $Y \equiv \rho(\omega + \rho)^{-1/(1+\mu)}$. The term $Y$ roughly depicts the change in money demand in the transaction cost model, relative to the CIA model. If $Y = 0$, one can easily verify that only corner solutions occur. Thus, modifying the cost of financing investment is insufficient to produce interior solutions; what is essential is the change in money demand $Y > 0$. Suppose that money growth rate decreases gradually. In the CIA case, at some instant, seigniorage resources become low enough, so that the income-tax is the only admissible way of government finance, to maximize growth. With an interest-elastic money demand, on the contrary, the more money growth rate decreases, the more money demand increases. Thus, seigniorage resources remain high enough not to be completely eliminated from the trade-off with taxes. Consequently, interior solutions emerge in the transaction cost model relative to the CIA case.

Of course, if seigniorage flight is high (high money multiplier $1/h$), the optimal trade-off yields a corner solution with no seigniorage, while for a low multiplier the optimal solution is the corner solution without taxes (see *Figure 6*). However, for intermediate values of $h$, the optimal strategy for government finance is a mix between income taxation and seigniorage.

*Figure 6a : Sensitivity Analysis (economic growth)*

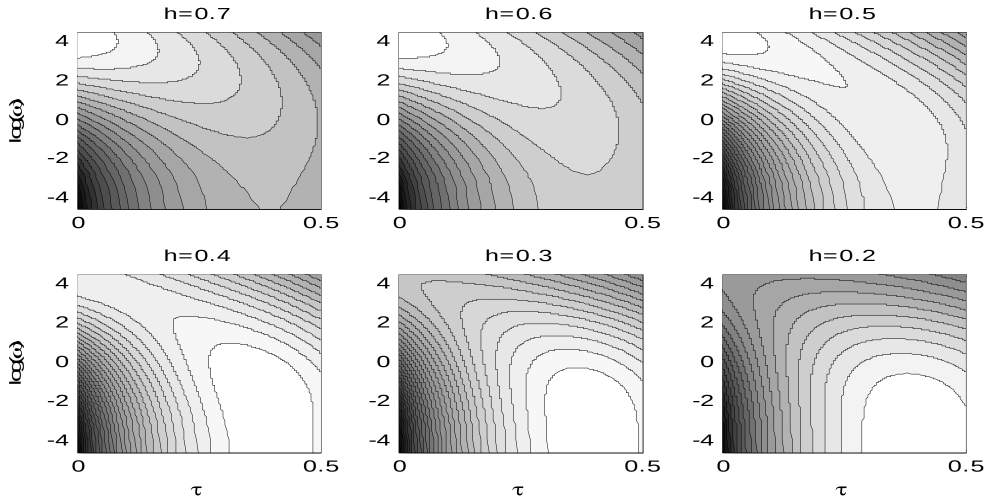
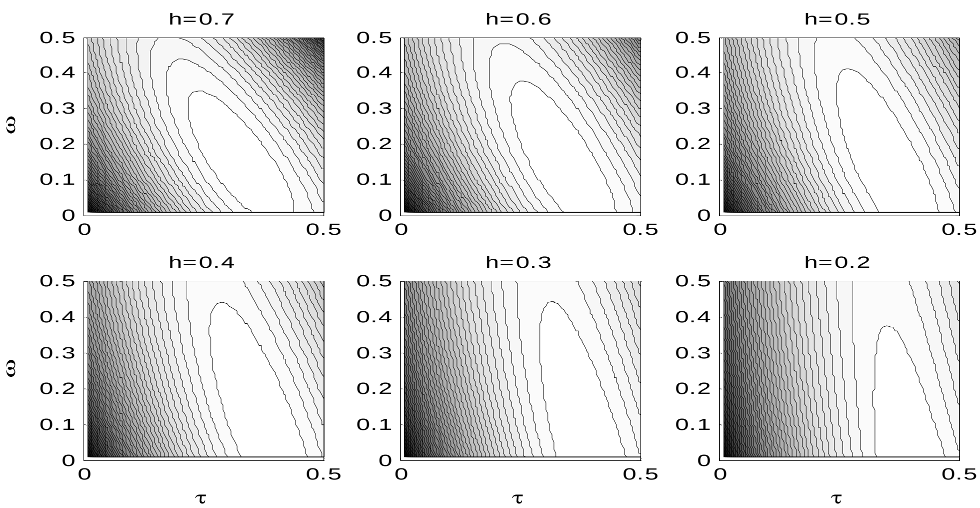



*Figure 6b : Sensitivity Analysis (welfare)*




*For simulation values:* $S = 1$, $\alpha = 0.6$; $\rho = 0.1$, $\delta = \theta = 0.05$, $\mu = 1$.

These results can easily be interpreted. In financially developed countries, seigniorage represents only a very small part of government resources, and it is more efficient to finance public spending exclusively by a flat tax rate on income. In our model, such countries must optimally choose low-inflation monetary policies, and government should mainly use income taxation. In countries with narrow financial markets, on the contrary, seigniorage is an essential resource for public finance, and governments may optimally choose "financial repression" as a way of finance. Thus, our model may explain why some governments resort to seigniorage and inflationary finance, and others rather to high tax-rates, as result of growth or welfare maximizing strategies in different structural environments, notably concerning the financial development level.

## 5. Empirical evidence

In this section, we propose several empirical findings to better support our theoretical results. We perform our analysis on a dataset of 110 available countries, with yearly data covering the period 1975-2000. We use the real output growth rate $\gamma$, the tax rate $\tau$ (computed as the ratio between fiscal revenues and output), the money growth rate $\omega$ (changes in narrow money, IFS line 34) and a variable for financial development $\tilde{h}$, defined as the ratio of bank deposits to output[20]. All variables are from two *International Monetary Fund* (IMF) datasets, namely *International Financial Statistics* (IFS) and *Government Finance Statistics* (GFS).

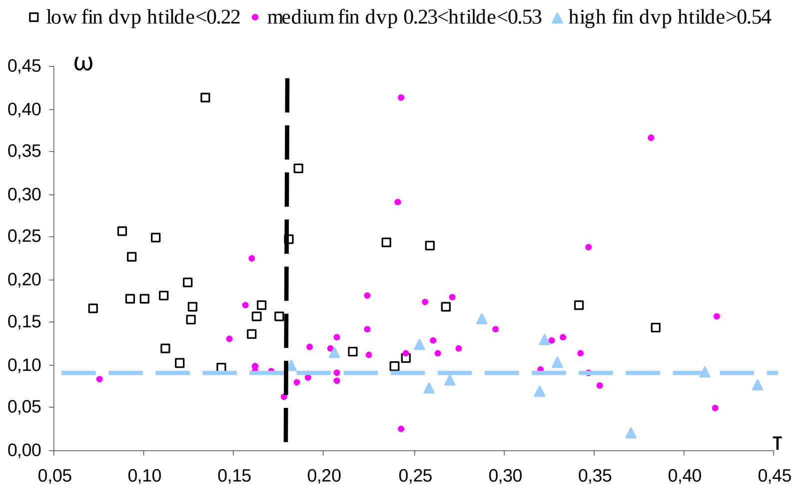
According to our model, the public finance stance critically depends on the financial development level, as countries with a "high" ("low") level of financial development use mainly taxes (seigniorage). Thus, our theoretical model supports that financial development is positively related to taxes and negatively related to seigniorage.

Let us first present some descriptive statistics. We compute average values for taxes, seigniorage and financial development, for each country over the period 1975-2000. Next, we classify countries in three groups: "low" financial development ($\tilde{h} < 0.22$) countries, "medium" financial development ($0.23 < \tilde{h} < 0.53$) countries and "high" financial development ($\tilde{h} > 0.54$) countries.[21]

[20] The variable $\tilde{h}$ lies between 0.1 and 1.65, and higher $\tilde{h}$ stands for higher financial development. Average values over the 1975-2000 period lie between 0.10 and 0.91.

[21] These values are selected to match our empirical results in the estimated structural equation (23) below. We obtain respectively 44 countries in the first group (low financial development), 51 in the second (medium financial development) and 15 countries in the third group (high financial development). To check robustness, we have considered different threshold values, without identifying qualitative changes in our results.

*Figure 7 – The trade-off between taxes and seigniorage according to financial development* $\tilde{h}$




In line with our model, an important number of countries use mainly one instrument. Countries with "low" financial development are mostly located on the left side of the vertical dotted line, with high seigniorage and low taxes, while countries with "high" financial development are mostly situated below the dotted horizontal line, with high taxes and low seigniorage. However, several countries with a "medium" developed financial system present fairly important levels of taxes and seigniorage, like in our transaction costs model of *section 4*.

We next propose several simple cross-section regressions:

*Figure 8 – Cross-section regressions between taxes, seigniorage and financial development*

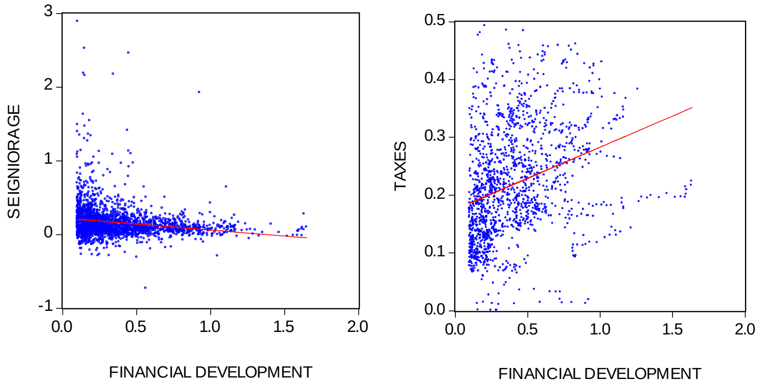



*Figure 8* confirms that the strategy that governments adopt in building public resources crucially depends on the financial structure of the economy, since higher financial development implies higher taxes and lower seigniorage.[22] To investigate the significance of the two *scatters*, *Table 2* reports the result of several regressions close to *Figure 8*.

[22] To check robustness, we excluded monetary growth rates superior to 300%, as the negative relation between financial development and money growth rate is even more robust, with the eliminated values included.

*Table 2 – The relation between financial development, taxes and seigniorage*

| | ***Dependent variable*** | | |
|---|---|---|---|
| | **Seigniorage [1]** | **Taxes [2]** | **Fin. Dvp. [3]** |
| constant | 0.223 (0.008)*** | 0.173 (0.004)*** | 0.279 (0.017)*** |
| financial development | -0.163 (0.017)*** | 0.118 (0.009)*** | |
| seigniorage | | | -0.259 (0.027)*** |
| taxes | | | 0.803 (0.067)*** |
| Observations | 2349 | 1561 | 1534 |
| Countries | 108 | 108 | 108 |
| Adjusted $R^2$ | 0.0365 | 0.1074 | 0.1401 |
| $F$ Fisher | 90.0 [0.00] | 187.7 [0.00] | 124.7 [0.00] |

***1% significance; standard error in parenthesis, p-value into brackets

The coefficients of the first two regressions are (highly) significant and in line with results in *Figure 8*. In particular, even if we exclude seigniorage values higher than one, the negative correlation in [1] is still significantly strong. Regression [3] shows that seigniorage and taxes are still significantly correlated with financial development, even when considered jointly.

Finally, we directly test a "structural" growth equation. We log-linearize the rate of economic growth in our transaction cost model (relation 22), so that:

$$\log(\gamma+\delta+\rho)=\beta_0+\beta_1\log(1-\tau)+\beta_2\log\left(\tau+\varphi h(X-Y)\right)+\beta_3\log(1+\varphi X) \qquad (23)$$

with $X\equiv(\omega+\rho)^{\frac{\mu}{1+\mu}}$ and $Y=\rho(\omega+\rho)^{\frac{-1}{1+\mu}}$.

To test relation (23), we use a transformed variable for financial development. Since our theoretical variable $h$ lies between 0 and 1 and is negatively linked to financial development, we define: $h\equiv 1-\tilde{h}/\tilde{h}^{\max}$, with $\tilde{h}^{\max}$ the highest level of financial development, *i.e.* $\tilde{h}^{\max}=1.65$.[23] Thus, low financial development countries $\left(\tilde{h}\approx 0.1\right)$ will have a $h$ value close to zero, and high financial development countries ($\tilde{h}$ approaching 1.65) will have a $h$ value close to one, as in our theoretical model. Results of the estimation of (22) using annual data for $\gamma$, $\tau$, $\omega$ and $h$ are reported in *Table 3*.

[23] Precisely, we must transform $\tilde{h}$ in order to *i*) reduce it to the interval $[0;1]$ and *ii*) inversely correlate it with the financial development level. Results are unaltered when using $h\equiv\exp\left(-\tilde{h}\right)$ or $h\equiv 1-\log\left(1+\tilde{h}\right)$, as shown in an *Appendix* available on request.

*Table 3 – The relation between taxes, seigniorage and growth*

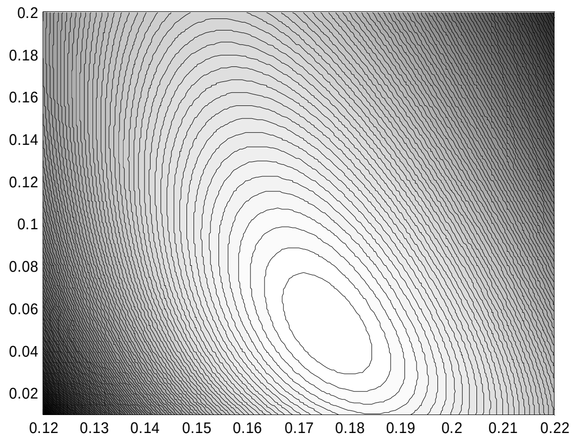
| | **log(growth)** |
|---|---|
| $\beta_0$ | -1.428 (0.086)*** |
| $\beta_1$ | 0.394 (0.109)*** |
| $\beta_2$ | 0.091 (0.032)*** |
| $\beta_3$ | -0.432 (0.193)*** |
| Observations | 1478 |
| Countries | 108 |
| Adjusted $R^2$ | 0.1693 |
| $F$ Fisher | 143.7 [0.00] |

***1% significance; standard error in parenthesis, p-value into brackets

For $\rho = 0.1$, $\delta = 0.05$, $\theta = 0.05$, $\mu = 1$

Remark first that all coefficients are significant and their sign in line with our model (positive for $\beta_1$ and $\beta_2$, negative for $\beta_3$). Using these coefficients, *Figure 9* below illustrates the existence of an interior growth solution, with active monetary and fiscal instruments:

*Figure 9 – Empirical evidence of the existence of an interior financing solution*




For $\rho = 0.1$, $\delta = 0.05$, $\theta = 0.05$, $\mu = 1$, $h = 0.5$

*Figure 9* confirms both the existence of an interior growth solution and the presence of thresholds in taxes and money to growth relation. For these parameters, we obtain an interior solution for $h \in [0.5;0.8]$. These margins roughly correspond to the ones we selected in *Figure 7* namely $\tilde{h} \in [0.22;0.53]$ after performing the transformation, and countries with financial development below (above) these values should compose their financing strategy of taxes (seigniorage) only. Notice also that the values of the instruments associated with interior solutions are quite similar in *Figure 7* and *Figure 9* respectively. The major difference with respect to our theoretical results comes from the estimated optimal tax rate, whose value corresponds to one half of its theoretical value, as a consequence of the fact that data for the tax rate lie between 0.01 and 0.49, with an average of 0.22 and median at 0.20.

## 6. Concluding remarks

In this paper, we develop the Barro (1990) model to money financing as an alternative to income taxation in government finance, in the spirit of Palivos & Yip (1995), but with endogenous productive public expenditures. With a CIA constraint on consumption only, money financing is always growth enhancing, since it does not distort the pattern of capital accumulation. If all transactions (including investment expenditures) are submitted to the CIA constraint, on the contrary, both income taxation and seigniorage distort capital accumulation. Since these two instruments provide resources for productive spending, our model exhibits thresholds between economic growth and both monetary and fiscal policies.

The presence of thresholds on both taxes and seigniorage allows some interesting comparisons between these two ways of government finance. Specifically, to maximize growth and/or welfare, some countries will use seigniorage only, and others taxes only, depending on the degree of financial development. Thus, our CIA model avoids the undesirable result of Palivos & Yip (1995), where seigniorage-only is always the growth-maximizing strategy. Furthermore, if this model is slightly modified in order to deal with transaction costs rather than with a CIA constraint, we are able to derive interior solutions for the optimal policy mix, with both seigniorage and tax rates, getting more realistic outcomes.

Empirical findings enforce our theoretical conclusions and highlight the significance of financial development for economic growth, in line with previous empirical evidences. For example, Fischer (1982) and Cukierman *et al.* (1992) show that seigniorage revenues are of high importance especially in developing countries. In addition, Giovannini & De Melo (1993) exhibit a high positive correlation between seigniorage and financial repression, and suggest that these two variables are complementary. Similarly, Boyd *et al.* (1997) report a negative correlation between inflation and indicators of financial development. Our model exhibits such a negative correlation, but as a result of maximizing growth or welfare policies. Although empirical evidence in the last section is still very crude, it confirms the robustness of our theoretical results, and highlights the need of further empirical research.

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## Appendix 1: Resolution of the CIA model

Let us write the *current Hamiltonian* as:

$$H_c = u(c_t) + \lambda_{1t}\left((1-\tau)y_t - c_t - \pi_t m_t - z_t + x_t\right) + \lambda_{2t}\left(z_t - \delta k_t\right) + \mu_t\left(m_t - c_t - \phi(z_t + g_t)\right)$$

where $\lambda_1$, $\lambda_2$ and $\mu$ are the costate variables associated respectively with $m_t$, $k_t$ and the CIA constraint. Notice that the ratio between $\mu_t$ and $\lambda_{1t}$ defines the nominal interest rate: $R_t = \mu_t / \lambda_{1t}$, as usual in CIA models. First order conditions are:

| | | |
|---|---|---|
| $/c_t$ | $u'(c_t) = \lambda_{1t} + \mu_t = \lambda_{1t}(1+R_t)$ | (A1a) |
| $/z_t$ | $\lambda_{2t}/\lambda_{1t} = 1 + \phi\mu_t/\lambda_{1t} = 1+\phi R_t$ | (A1b) |
| $/m_t$ | $\dot{\lambda}_{1t}/\lambda_{1t} = \rho + \pi_t - \mu_t/\lambda_{1t} = \rho - r_t$ | (A1c) |
| $/k_t$ | $\dot{\lambda}_{2t}/\lambda_{2t} = \rho + \delta - \lambda_{1t}(1-\tau)f'(k_t)/\lambda_{2t} = \rho + \delta - (1-\tau)f'(k_t)/(1+\phi R_t)$ | (A1d) |

As usual with a CIA constraint on consumption, the nominal interest rate introduces a wedge between the marginal utility of consumption and the shadow price of wealth $(\lambda_{1t})$ in (A1a). If $\phi > 0$, the rate of return of investment must also be deflated by the cost of financing $(1+\phi R_t)$ in (A1d). In addition, the CIA constraint on investment introduces a wedge between the real return of wealth (the real interest rate $r_t$ in (A1c)) and the net return of investment ($(1-\tau)f'(k_t)/(1+\phi R_t) - \delta$ in (A1d)), since wealth does not allow acquiring capital goods directly. Note that this wedge disappears if $\phi = 0$, as $\lambda_{1t} = \lambda_{2t}$ in (A1d). Thus, the Keynes-Ramsey relation is $\dot{c}_t / c_t = -S\left[\dot{\lambda}_{1t}/\lambda_{1t} + \dot{R}_t/(1+R_t)\right]$.

Goods market equilibrium yields the *IS* curve $\dot{k}/k = (g/k)^{1-\alpha} - (c/k) - (g/k) - \delta$, and money market equilibrium provides: $\dot{m}_t / m_t = \omega - \pi_t$. By differentiating (A4) and (A5) and defining intensive variables $c_k \equiv c/k$, $g_k \equiv g/k$ and $m_k \equiv m/k = c_k + \phi\left((\dot{k}/k) + g_k\right)$, we obtain a reduced form of the representative agent program in a five equations for five variables system (time indexes are hereafter omitted):

| | |
|---|---|
| $\dot{c}_k / c_k = S\left[r - \rho - \dot{R}/(1+R)\right] - \left(g_k^{1-\alpha} - c_k - g_k - \delta\right)$ | (A2a) |
| $\dot{R} = \left[(1+\phi R)(r+\delta) - \alpha(1-\tau)g_k^{1-\alpha}\right]/\phi$ | (A2b) |
| $g_k = \tau g_k^{1-\alpha} + h\omega m_k$ | (A2c) |
| $\dot{m}_k / m_k = \omega + r - R - g_k^{1-\alpha} + g_k + c_k + \delta$ | (A2d) |
| $m_k = \phi g_k^{1-\alpha} + (1-\phi)c_k$ | (A2e) |

We find the steady-state endogenous growth solution by imposing $\dot{c}_k = \dot{m}_k = \dot{g}_k = \dot{R} = 0$, which implies constant $c_k$, $m_k$, $g_k$ and $R$ values, so that initial variables ($c$, $k$, $g$, $m$) grow at the same constant rate $\gamma$, while $R$, $r$ and $\pi$ are constant. In steady-state, system (A2) gives rise to equations (7)-(8) of the main text. Notice that there are only “jump” variables in (A2), thus our model lacks any transitional dynamics. *Appendix 5* shows that the steady-state is stable.

## Appendix 2: Second order conditions for a maximum

A necessary condition for $\left(\overline{\overline{\tau}},\overline{\overline{\omega}}\right)$ to be an optimum is[24] $\gamma^*_{\tau\tau}\left(\overline{\overline{\tau}},\overline{\overline{\omega}}\right)\gamma^*_{\omega\omega}\left(\overline{\overline{\tau}},\overline{\overline{\omega}}\right)-\left(\gamma^*_{\tau\omega}\left(\overline{\overline{\tau}},\overline{\overline{\omega}}\right)\right)^2>0$. First order conditions are:

$$\gamma^*_\omega \equiv \frac{\partial\gamma^*(\tau,\omega)}{\partial\omega}=\frac{(1-\tau)(\tau+h\omega)^{\frac{1-2\alpha}{\alpha}}\left[(1-\alpha)h(1+\omega+\rho)-\alpha(\tau+h\omega)\right]}{(1+\omega+\rho)^2}=0\text{, and:}$$

$$\gamma^*_\tau \equiv \frac{\partial\gamma^*(\tau,\omega)}{\partial\tau}=\frac{(\tau+h\omega)^{\frac{1-2\alpha}{\alpha}}\left[(1-\alpha)(1-\tau)-\alpha(\tau+h\omega)\right]}{(1+\omega+\rho)}=0$$

Let us compute the second derivatives: $\gamma^*_{\omega\omega}\equiv\frac{\partial^2\gamma^*\left(\overline{\overline{\tau}},\overline{\overline{\omega}}\right)}{\partial\omega^2}=-\frac{\left(1-\overline{\overline{\tau}}\right)\left(\overline{\overline{\tau}}+h\overline{\overline{\omega}}\right)^{\frac{1-2\alpha}{\alpha}}(2\alpha-1)}{\left(1+\overline{\overline{\omega}}+\rho\right)^2}$, $\gamma^*_{\tau\tau}\equiv\frac{\partial^2\gamma^*\left(\overline{\overline{\tau}},\overline{\overline{\omega}}\right)}{\partial\tau^2}=-\frac{\left(\overline{\overline{\tau}}+h\overline{\overline{\omega}}\right)^{\frac{1-2\alpha}{\alpha}}}{\left(1+\overline{\overline{\omega}}+\rho\right)}<0$ and $\gamma^*_{\tau\omega}\equiv\frac{\partial^2\gamma^*\left(\overline{\overline{\tau}},\overline{\overline{\omega}}\right)}{\partial\tau\partial\omega}=-\frac{\alpha h\left(\overline{\overline{\tau}}+h\overline{\overline{\omega}}\right)^{\frac{1-2\alpha}{\alpha}}}{\left(1+\overline{\overline{\omega}}+\rho\right)}<0$. Using $h\left(1+\overline{\overline{\omega}}+\rho\right)=\left(1-\overline{\overline{\tau}}\right)$, one can easily verify that:

$$\gamma^*_{\tau\tau}\left(\overline{\overline{\tau}},\overline{\overline{\omega}}\right)\gamma^*_{\omega\omega}\left(\overline{\overline{\tau}},\overline{\overline{\omega}}\right)-\left(\gamma^*_{\tau\omega}\left(\overline{\overline{\tau}},\overline{\overline{\omega}}\right)\right)^2=-\frac{(1-\alpha)^2h\left(1-\overline{\overline{\tau}}\right)\left[\left(\overline{\overline{\tau}}+h\overline{\overline{\omega}}\right)^{\frac{1-2\alpha}{\alpha}}\right]^2}{\left(1+\overline{\overline{\omega}}+\rho\right)^3}<0.$$

Thus, the $\left(\overline{\overline{\tau}},\overline{\overline{\omega}}\right)$ couple cannot be an optimum.

## Appendix 3: The model with transaction costs

With transaction costs, the representative agent budget constraint is $\dot{k}_t+\dot{m}_t=(1-\tau)y_t-\psi(c_t+z_t+g_t,m_t)y_t-c_t-\delta k_t-\pi_t m_t+x_t$, replacing (4)-(5) in the main text. *Current Hamiltonian* and first order conditions are:

$$H_c=u(c_t)+\lambda_{1t}\left[(1-\tau)f(k_t,g_t)-\psi(c_t+z_t+g_t,m_t)y_t-c_t-z_t-\pi_t m_t+x_t\right]+\lambda_{2t}(z_t-\delta k_t)$$

$$/c_t \qquad u_c(c_t)=\lambda_{1t}(1+\psi_c y_t) \qquad \text{(A3a)}$$

$$/z_t \qquad \lambda_{2t}=\lambda_{1t}(1+\psi_z y_t) \qquad \text{(A3b)}$$

$$/m_t \qquad \dot{\lambda}_{1t}/\lambda_{1t}=\rho+\pi_t+\psi_m y_t=\rho+\pi_t-R_t=\rho-r_t \qquad \text{(A3c)}$$

$$/k_t \qquad \dot{\lambda}_{2t}/\lambda_{2t}=\rho+\delta-\left[(1-\tau)f_k(k_t,g_t)\right]\lambda_{1t}/\lambda_{2t} \qquad \text{(A3d)}$$

These relations have a standard interpretation. $\lambda_1$ is the shadow price of wealth, which differs from the shadow price of capital $(\lambda_2)$ in equation (A3b), since investment expenditures are subject to a transaction cost ($\psi_z y_t$ is the marginal transaction cost on investment expenditures). Similarly, in equation (A3a), the marginal utility of consumption has to be

[24] We use the notation: $x_y=\frac{\partial x(.)}{\partial y},\forall x,y$

distinguished from the shadow price of wealth, since there is a transaction cost on consumption expenditures (with $\psi_c y_t$ the marginal transaction cost on consumption expenditures). Equation (A3c) states that the marginal cost of money (the nominal interest rate $R_t$) must equalizes its marginal return ($-\psi_m y_t$, since money economizes transaction costs).

Using $\varphi \equiv \theta^{1/(1+\mu)}$, these first order conditions conduct to:

$$R_t = \theta(y_t / m_t)^{1+\mu} \tag{A4a}$$

$$u'(c_t) = \lambda_{1t}\left[1+\varphi(R_t)^{\frac{\mu}{1+\mu}}\right] = \lambda_{2t} \tag{A4b}$$

$$\dot{\lambda}_{2t} / \lambda_{2t} = \rho + \delta - \frac{(1-\tau)\, f'(k_t, g_t)}{1+\varphi(R_t)^{\frac{\mu}{1+\mu}}} \tag{A4c}$$

It clearly appears that the CIA constraint is a special case of the transaction technology (21). Effectively, if $\mu \to \infty$, then $y_t \to m_t$ and $\varphi \to 1$, and relations (A4b) and (A4c) become (A1ac) and (A1d) respectively, without any change in (A1b) and (A1c). The advantage of the transaction technology (21) is that it is more general, and that it gives rise to an interest-elastic money demand. Effectively, from (A4a), we obtain a simple usual form for money demand:

$$m_t = \varphi y_t \,(R_t)^{\frac{-1}{1+\mu}} = m\Big(\underset{+}{y_t}, \underset{-}{R_t}\Big) \tag{A5}$$

Differentiating (A3a) and (A3b) we obtain the reduced form of the model:

$$\dot{c}_k / c_k = S\left[\alpha(1-\tau)q(R)g_k^{1-\alpha} - \delta - \rho\right] - \left(g_k^{1-\alpha} - g_k - c_k - \delta\right) \tag{A6}$$

$$\dot{R} / R = (1+\mu)\left[r + \delta - \alpha(1-\tau)q(R)g_k^{1-\alpha}\right] / \left[\mu(1-q(R))\right] \tag{A7}$$

with $q(R_t) = \left[1+\varphi(R_t)^{\frac{\mu}{1+\mu}}\right]^{-1}$. The money to capital ratio is: $m_k = \varphi g_k^{1-\alpha}\,(R)^{\frac{-1}{1+\mu}}$, and we can find the public spending to capital ratio in the government budget constraint: $g_k = \tau g_k^{1-\alpha} + h\omega m_k$, namely: $g_k = \left[\tau + \varphi h\omega(R)^{\frac{-1}{1+\mu}}\right]^{1/\alpha}$. Differentiating this expression with respect to time, we get: $\frac{\dot{g}_k}{g_k} = \frac{-1}{\alpha(1+\mu)} \frac{\varphi h\omega R^{\frac{-\mu}{1+\mu}}}{\tau + \varphi h\omega R^{\frac{-1}{1+\mu}}} \dot{R}$, and the real interest rate can be obtained from the money market equilibrium $\dot{m}/m = \omega + r - R$. Since: $\dot{m}_k / m_k = (1-\alpha)\dot{g}_k / g_k - \dot{R}/[(1+\mu)R] = \dot{m}/m - \dot{k}/k$, we find: $r = \dot{m}/m + R - \omega = (1-\alpha)\dot{g}_k / g_k - \dot{R}/[(1+\mu)R] + \left(g_k^{1-\alpha} - g_k - c_k - \delta\right) + R - \omega$. One can verify that the dynamic properties of the model are unchanged: we still obtain two positive eigenvalues, so there are no transitory dynamics.

The long-run solution of the model can still be described by two relations between $\gamma$ and $g_k$. The first relation is the Keynes-Ramsey relation in equilibrium $\left(\dot{R}=0\right)$: $\gamma^* = S\left(r^* - \rho\right)$, with: $r^* = \alpha(1-\tau)q(R^*)(g_k^*)^{1-\alpha} - \delta$ and $R^* = \omega + r^* - \gamma^* = \omega + \rho - \gamma^*(S-1)/S$ in steady-state. Thus: $g_k^A(\gamma) = \left\{ \frac{1}{\alpha(1-\tau)} \left( \frac{\gamma^*}{S} + \delta + \rho \right) \left( 1 + \varphi\left(\omega + s(\gamma^*)\right)^{\frac{\mu}{1+\mu}} \right) \right\}^{\frac{1}{1-\alpha}}$. The second relation comes from the government budget constraint: $g_k = \left[ \tau + \varphi h \omega R^{\frac{-1}{1+\mu}} \right]^{1/\alpha}$ with $R = \omega + \rho - \left( \frac{S-1}{S} \right)\gamma$ in steady-state, thus: $g_k^B(\gamma) = \left[ \tau + \varphi \omega h \left( \omega + s(\gamma^*) \right)^{\frac{-1}{1+\mu}} \right]^{\frac{1}{\alpha}}$, and we find the long-run solution by equalizing $g_k^A(\gamma)$ with $g_k^B(\gamma)$. The implicit relation defining long-run economic growth (equation (12) in the main text) is simply replaced with:

$$\left(\gamma^*/S + \delta + \rho\right)\left[1 + \phi_1\left(\omega + s(\gamma^*)\right)^{\mu/(1+\mu)}\right] = \alpha(1-\tau)\left[\tau + \phi_0 h\omega\left(\omega + s(\gamma^*)\right)^{-1/(1+\mu)}\right]^{(1-\alpha)/\alpha} \qquad \text{(A8)}$$

and we find (12) of the main text if $\mu \to \infty$. If $S = 1$, we get (22) in the text, and, using the same reasoning as for the CIA case, we obtain an analytical form of $\gamma^*$ in the general case $S \neq 1$.

**Technical Appendix (not for publication)**

**Appendix 4: A CIA constraint on consumption and investment only**

In this Appendix, we introduce a more general version of the CIA constraint, namely: $c+\beta g+\phi z=m$. The case $\beta=\phi$ corresponds to the main text, while the case $\beta=0$ corresponds to a CIA constraint on consumption and private investment expenditures only, as in Stockman (1981). This new constraint does not modify first order conditions, as $g$ is exogenous for households. Equation (7) is unchanged, but equation (8) becomes:

$$g_k=\frac{(\tau+h\omega)\,g_k^{1-\alpha}-(1-\phi)\,h\omega(\gamma+\delta)}{1+(1-\beta)\,h\omega} \tag{8'}$$

If $\beta=0$ and $\phi=1$, as in Stockman (1981), only the definition of the public spending ratio must be modified: $g_k=\left(\frac{\tau+h\omega}{1+h\omega}\right)^{1/\alpha}$, in place of (8), without change in qualitative results.

One interesting situation is that described by Palivos & Yip (1995), namely a CIA constraint on consumption and a part of private investment: $\beta=0$ and $\phi\in[0,1]$. The long-run economic growth rate is still obtained from (7) and (8'); but, in our setup with *productive* public spending, it is rather difficult to derive first and second order conditions for the maximization of economic growth. So, we have performed some numerical simulations. The main results are:

i) concerning long-run growth maximization, there is no interior maximum, all maxima are corner solutions with only one active instrument,
ii) when $h=1$, the maximizing-growth strategy is always a seigniorage only strategy, as in Palivos & Yip (1995), whatever the value of $\phi$,
iii) for $0<h<1$, high (low) $\phi$ values conduct to taxes (seigniorage) only strategies, as higher $\phi$ implies more distortion of seigniorage on private investment, making taxes more attractive, while lower $\phi$ implies less distortion, down to the limiting case $\phi=0$ (CIA on consumption only, no distortion on capital accumulation),
iv) for $0<\phi\leq 1$, high (low) $h$ values conduct to seigniorage (taxes) only strategies, since government is able to retrieve an important (a small) part of total seigniorage,

Comparing results ii), iii) and iv), we can see that coefficient $h$ in our model plays a similar role as coefficient $\phi$ in Palivos & Yip (1995) model. Furthermore, since the maximizing-growth strategy when $h=1$ (seigniorage only) is independent of the value of $\phi$ (point ii), while the maximizing-growth strategy when $\phi=1$ depends on the value of $h$ (point iv), considering alternative values of $h$ is more general than considering alternative value of $\phi$. This explains why we fix $\phi=1$ in the main text.

**Appendix 5: Stability of the steady-state**

To study transitory dynamics, we use the more general form of the CIA constraint of *Appendix 4*, and we consider three cases: a CIA constraint on consumption only $(\beta=\phi=0)$, a CIA constraint on consumption and private investment only $(\beta=0,\ \phi=1)$ and a constraint over consumption, private and public investment $(\beta=\phi=1)$. Let us bring the two latest cases together, by studying the more general case $\beta\in[0;1]$ and $\phi=1$.

*The case* $\beta=\phi=0$

The reduced form of the model can be written in two differential equations in $g_k$ and $R$:

$$\dot{g}_k=\frac{h\omega m_k\left(\omega+r-R-\left(g_k^{1-\alpha}-g_k-m_k-\delta\right)\right)}{1-\tau(1-\alpha)g_k^{-\alpha}}$$
$$\dot{R}=\frac{(1+R)}{S}\left[S(r-\rho)+R-r-\omega\right]$$

Since investment is not affected by the CIA constraint, the real interest rate equals the marginal product of capital: $r=\alpha(1-\tau)g_k^{1-\alpha}-\delta$, and we find the money to capital ratio in the government budget constraint : $m_k=\left(g_k-\tau g_k^{1-\alpha}\right)/(h\omega)$. The *Jacobian* matrix evaluated in the neighborhood of the steady-state values $\left(g_k^*,R^*\right)$ is:

$$J=\begin{pmatrix}\dfrac{M^*\left(h\omega r_{g_k}^*+M_{g_k}^*\right)}{h\omega M_{g_k}^*} & -\dfrac{M^*}{M_{g_k}^*}\\ -\dfrac{(S-1)^2r_{g_k}^*}{S} & \dfrac{1+R^*}{S}\end{pmatrix}$$

with: $r_{g_k}\equiv\dfrac{dr}{dg_k}=\alpha(1-\alpha)(1-\tau)\left(g_k^*\right)^{-\alpha}>0$, $M^*=g_k^*-\tau\left(g_k^*\right)^{1-\alpha}=h\omega c_k^*>0$ and $M_{g_k}^*\equiv\dfrac{dM}{dg_k}=1-\tau(1-\alpha)\left(g_k^*\right)^{-\alpha}>\dfrac{M^*}{g_k^*}>0$.

The *Trace* of $J$ is obviously positive and its *Determinant* is: $Det=\dfrac{M^*}{Sh\omega M_{g_k}^*}\left[\left(h\omega r_{g_k}^*+M_{g_k}^*\right)-h\omega(S-1)^2r_{g_k}^*+R^*\left(h\omega r_{g_k}^*+M_{g_k}^*\right)\right]$ which is positive if $S$ is not "very high", namely $S<\bar{S}$, which we suppose (notice that $\bar{S}>2$). This means that both eigenvalues are positive. As $g_k$ and $R$ are jump variables, Blanchard-Kahn conditions imply that there are no transitional dynamics and that the economy jumps from a steady-state to another following any impulse on parameters.

*The case* $\beta\in[0;1]$ *and* $\phi=1$

In this case, the reduced form can be expressed in $c_k$ and $R$:

$$\frac{\dot{c}_k}{c_k}=S\left(r-\rho-\frac{\dot{R}}{1+R}\right)-\left(g_k^{1-\alpha}-g_k-c_k-\delta\right)$$
$$\dot{R}=(1+R)(r+\delta)-\alpha(1-\tau)g_k^{1-\alpha}$$

This time, the government constraint together with the CIA constraint provide the public spending to capital ratio: $g_k = \left( \frac{\tau + h\omega}{1 + h\omega(1-\beta)} \right)^{1/\alpha}$, which is a constant. The real interest rate comes from money equilibrium $(\dot{m}/m = \omega + r - R)$. Since $m_k = g_k^{1-\alpha} - (1-\beta) g_k$ and $g_k$ is constant, we have: $\dot{m}/m = \dot{k}/k$ and $r = \left( g_k^{1-\alpha} - g_k - c_k - \delta \right) + R - \omega$. The *Jacobian* matrix is:

$$J = \begin{pmatrix} c_k^* & -\frac{c_k^* S \left( r^* + \delta \right)}{1 + R^*} \\ -\left(1 + R^*\right) & 1 + R^* + r^* + \delta \end{pmatrix}$$

The *Trace* of $J$ is positive and its *Determinant* is: $Det = c_k^* \left[ 1 + \omega + (1 - S)(\gamma^* + \rho + \delta) \right]$, with $\gamma^*$ the steady-state growth rate. As before, we suppose that $S < \overline{\overline{S}}$, with $\overline{\overline{S}} > 2$, with $\overline{\overline{S}}$ the highest elasticity of substitution so that the determinant is positive. As in the previous case, both eigenvalues are positive and there are no transitional dynamics, since $c_k$ and $R$ are jump variables. Notice that dynamics is qualitatively unaffected by the value of $\beta$.